INP Report 1666/PH (1994)

Gluons from logarithmic slopes of F_2 in the NLL approximation

K.Golec–Biernat *

Department of Theoretical Physics
Institute of Nuclear Physics
Radzikowskiego 152, Kraków, Poland

Abstract

We make a critical, next–to–leading order, study of the accuracy of the "Prytz" relation, which is frequently used to extract the gluon distribution at small x from the logarithmic slopes of the structure function F_2. We find that the simple relation is not genarally valid in the HERA regime, but show that it is a reasonable approximation for gluons which are sufficiently singular at small x.

*e–mail address: golec@bron.ifj.edu.pl

Recently a new method of determination of the gluon distribution in the nucleon from the deep-inelastic scattering (DIS) data was proposed [1]. The method is based on an approximate relation between the gluon density and the logarithmic slopes of the nucleon structure function F_2, derived using the leading logarithmic approximation (LLA) formulae. The crucial point in the derivation is the observation that for the existing parametrizations of the parton distributions the logarithmic slope of F_2 at small x ($x < 10^{-2}$) depends mostly on the gluon density and the sea quarks contribution can be neglected. In this case the following formula for the logarithmic slope of F_2 is valid (for $n_f = 4$)

$$\frac{dF_2}{dlogQ^2} \approx \frac{5\alpha_s}{9\pi} \int_x^1 dz\, G(\frac{x}{z}, Q^2)\; P_{qg}(z)\;, \tag{1}$$

where $P_{qg}(z) = (1-z)^2 + z^2$ is the Altarelli–Parisi splitting function in LLA.

The integral in (1) was approximated by the value of the gluon distribution at the point $2x$, and the final relation was found

$$\frac{dF_2}{dlogQ^2}(x) \approx \frac{10\alpha_s}{27\pi}\; G(2x)\;. \tag{2}$$

to be valid at small x below 10^{-2}.

Relation (2) was used by the H1 colaboration at DESY to estimate the gluon distribution in the proton from the first measurement of the structure function F_2 in the DIS region at the e-p colider HERA at DESY[2]. The slope at l.h.s of relation (2) was determined at $Q^2 = 20\ GeV^2$ from the data and the gluon distribution was found to exhibit strong rise when $x \to 0$.

Relation (2) helps to estimate the gluon distribution in LLA, while most of existing predictions of the small x behaviour of the parton distributions are done in the next-to-leading logarithmic approximation (NLLA). It means that the corrections which are one order higher in α_s are taken into account both in the Altarelli–Parisi equations and in the structure functions to determine the parton distributions. A natural question arises whether relation (2) remains a good approximation also for NLLA gluon ditributions. The author of paper [1] claims that this relation is valid with an accuracy of around 20 %, taking into account many uncertainties, also those resulting from higher order α_s corrections. We will show that this is not generally true in NLLA. The validity of relation (2) crucially depends on a type of gluon distribution.

In order to show this we computed the logarithmic slope of F_2 on l.h.s of formula (2) for three different sets of existing parton distribution parametrizations at small x ; MRS parton distributions D'_- and D'_- [3] , and GRV ones [4]. All of them are extrapolations from the region of high x, where the fixed targed DIS data exist. These distributions represent the whole spectrum of possible behaviours of the sea quarks and gluon distributions at small x. The Lipatov motivated D'_- parametrization predicts strong rise of gluon and sea, proportional to $x^{-0.5}$ when $x \to 0$, whereas D'_0 distributions are flat. They tend to constant values in the limit of small x. The gluon and sea distributions from the GRV set lie in between. They are much steeper than D'_0 distributions but not as steep as D'_- ones. We computed the slope of F_2 numerically, using a part of a computer program, which solves the Altarelli–Parisi equations in NLLA, prepared to analyse the DIS data from HERA [5]. Having done that we compared the computed exact slopes of F_2 to r.h.s of relation (2), which can easily be found for the parton parametrizations under consideration.

The results of this analysis are shown in **Fig.1**. We plot there the exact and approximate slopes (r.h.s of relation (2)) computed at $Q^2 = 20\ GeV^2$, as well as their ratios. It is easy to see

that the validity of relation (2) in NLLA depends strongly on the parton distributions. While for the steep gluons from the D'_- set the relation is still approximately true, then for the flat type $D_0^{'}$ gluons it is dramatically violated. For the GRV parametrization relation (2) could be acceptable but for x bigger then 10^{-4}.

In order to clarify the results from **Fig.1** we computed contributions of different terms to the exact NLLA formula for the logarithmic slope of F_2. This formula has a form of (1), where the NLLA splitting function $P_{qg}^{(1)}$ [6] was added to LLA one

$$P_{qg}(z) \rightarrow P_{qg}(z) + \frac{\alpha_s}{2\pi} P_{qg}^{(1)}(z) \tag{3}$$

In addition, there are also terms with quark distributions with LLA and NLLA contributions in the exact formula for the slope. In **Fig.2** we plotted all four contributions to $dF_2/dlogQ^2$ at $Q^2 = 20\ GeV^2$, coming from the gluon (*on the left*) and quark (*on the right*) terms computed in LL (*dashed lines*) and NLL (*dotted lines*) approximations. The exact slopes (*continuous lines*) are also shown. The computation was performed for the parton distribution sets under consideration. We immediately see that the quark contributions in both approximations can be neglected in the derivation of the approximate relation. The slope of F_2 is determined mostly by the gluon terms. For the singular MRS($D_-^{'}$) gluons the LLA contribution dominates, while for the flat MRS($D_0^{'}$) ones the NLLA contibution is as important as the LLA one. This is the reason why the approximate relation (2) is not valid in NLLA for the flat type gluon distributions. In such case relation (2) applied to extract gluon from data may overestimate the steepness of the gluon distribution. We illustrate the last statement in **Fig.3**, where the exact gluons used to compute the slope of F_2 and the gluons estimated from relation (2) are plotted.

In conclusion, the simple relation (2) between the logarithmic slope of F_2 and the gluon distribution is not generally valid in the NLL approximation. It remains a reasonable approximation only for gluons which are sufficently singular at small x.

Fortunately, the gluons estimated by the H1 collaboration from the first data seem to be steep enough to exclude the case with flat-like gluons. Nevertheless, the problem of determination of the gluon distribution at small x is much deeper from the theoretical point of view. We have discussed this problem within the standard approach based on the Altarelli–Parisi evolution equations and the standard factorization formula. A different, more appropriate at small x approach was proposed [7, 8]. It is based on the Lipatov equation and a new factorization theorem. This approach may lead in principle to different results from those based on the Altarelli–Parisi equations [9]. A future analysis of new data from HERA should clarify these problems.

Acknowledgment

I would like to thank J. Feltesse, M.W. Krasny, J. Kwieciński, A.D.Martin and J.Turnau for helpful comments and discussions. The EC "Go West" fellowship is gratefully acknowledged. This work has been partially supported by the Polish Committee for Scientific Research under the grant 2 0198 91 01.

References

[1] K.Prytz, Phys. Lett. **311B** (1983) 286.

[2] H1 Collab., I.Abt. et al., Phys.Lett. **B321** (1994) 161,

[3] A.D.Martin, W.J.Stirling and R.G.Roberts, Phys.Lett. **306B** (1993) 145.

[4] M.Gluck, E.Reya and A.Vogt: DO-TH 91/07: see also Z. Phys. **C48** (1990) 471.

[5] K.Golec–Biernat, W.Krasny, S.Riess, in preparation.

[6] W.Furmański and R.Petronzio, Phys.Lett. **B97** (1980) 437, Z. Phys. **C11** (1982) 263.

[7] S.Catani, F.Fiorani and G.Marchesini, Phys.Lett. **234B** (1990) 339; Nucl.Phys.**B357** (1991) 167, S.Catani, M.Ciafaloni and F.Hautman Phys.Lett. **B242** (1990) 97; Nucl.Phys. **B366** (1991) 135.

[8] A.J.Askew, J. Kwieciński, A.D.Martin and P.Sutton, Phys.Rev. **D47** (1993) 3775, Mod. Phys. Lett. **A8** (1993) 3813, Durham preprint DTP/93/28 to appear in Phys.Rev.D.

[9] A.J.Askew, K.Golec-Biernat, J. Kwieciński, A.D.Martin and P.Sutton, IFJ preprint 1653/PH, Krakow (1993) to appear in Phys.Lett. B

Figure Captions

Fig. 1 Exact slopes computed in NLLA (continuous lines) versus slopes resulting from the approximate Prytz relation assumed to be still valid in NLLA (dashed lines). The slopes are computed for MRS(D'_-), MRS(D'_0) and GRV parton ditributions. On the right hand side the ratio of the approximate to exact slopes is shown. All quantities are computed at $Q^2 = 20\ GeV^2$.

Fig. 2 Parton contributions to the exact slope $dF_2/dlogQ^2$ (continuous lines) at $Q^2 = 20\ GeV^2$ computed for parton distributions under consideration in NLLA. Contibution from gluons is shown on the left and from quarks on the right. Dashed and dotted lines show the leading log and next–to–leading contribution respectively .

Fig. 3 Comparison of the exact gluons used to compute the exact slopes of F_2 in NLLA (continuous line) to those estimated from the approximate Prytz relation (dashed line) at $Q^2 = 20\ GeV^2$. The ratios of the approximate gluon distributions to the exact ones are shown on the right hand side.